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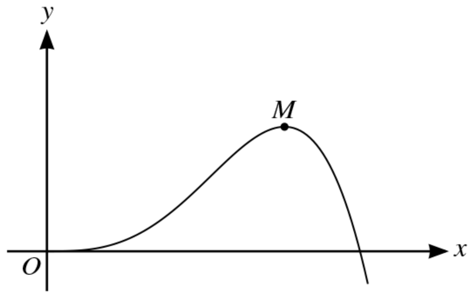
MATHEMATICS (9709) P2

TOPIC WISE QUESTIONS + ANSWERS | COMPLETE SYLLABUS

Chapter 3

Trigonometry

79. 9709_s20_qp_21 Q: 5



The diagram shows part of the curve with equation $y = x^3 \cos 2x$. The curve has a maximum at the point M.

(a) Show that the x-coordinate of M satisfies the equation $x = \sqrt[3]{1.5x^2 \cot 2x}$. [3]

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(b) Use the equation in part **(a)** to show by calculation that the x-coordinate of M lies between 0.59 and 0.60. [2]

(c) Use an iterative formula, based on the equation in part **(a)**, to find the x-coordinate of M correct to 3 significant figures. Give the result of each iteration to 5 significant figures. [3]

80. 9709_s20_qp_22 Q: 6

The polynomial $p(x)$ is defined by

$$p(x) = 6x^3 + ax^2 - 4x - 3,$$

where a is a constant. It is given that $(x + 3)$ is a factor of $p(x)$.

(a) Find the value of a. [2]

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(b) Using this value of a, factorise $p(x)$ completely. [3]

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(c) Hence solve the equation $\mathrm{p}(\operatorname{cosec}\theta) = 0$ for $0° < \theta < 360°$. [3]

81. 9709_w20_qp_21 Q: 6

It is given that $3\sin 2\theta = \cos\theta$ where θ is an angle such that $0° < \theta < 90°$.

(a) Find the exact value of $\sin\theta$. [2]

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(b) Find the exact value of $\sec\theta$. [2]

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(c) Find the exact value of $\cos 2\theta$. [2]

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82. 9709_w20_qp_22 Q: 1

Solve the equation $7\cot\theta = 3\operatorname{cosec}\theta$ for $0° < \theta < 90°$. [3]

83. 9709_m19_qp_22 Q: 1

Solve the equation $\sec^2\theta + \tan^2\theta = 5\tan\theta + 4$ for $0° < \theta < 180°$. Show all necessary working. [4]

84. 9709_s19_qp_21 Q: 7

(i) Show that $2\,\mathrm{cosec}\,2\theta\cot\theta \equiv \mathrm{cosec}^2\theta$. [3]

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(ii) Hence show that $\mathrm{cosec}^2 15^\circ \tan 15^\circ = 4$. [2]

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(iii) Solve the equation $2\operatorname{cosec}\phi\cot\frac{1}{2}\phi + \operatorname{cosec}\frac{1}{2}\phi = 12$ for $-360° < \phi < 360°$. Show all necessary working. [5]

85. 9709_s19_qp_22 Q: 7

(a) **(i)** Express $4\sin\theta + 4\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(ii) Hence find the smallest positive value of θ satisfying the equation $4\sin\theta + 4\cos\theta = 5$. [2]

(b) Solve the equation

$$4\cot 2x = 5 + \tan x$$

for $0 < x < \pi$, showing all necessary working and giving the answers correct to 2 decimal places. [6]

86. 9709_w19_qp_21 Q: 6

(a) Showing all necessary working, solve the equation

$$\sec \alpha \operatorname{cosec} \alpha = 7$$

for $0° < \alpha < 90°$. [5]

(b) Showing all necessary working, solve the equation

$$\sin(\beta + 20°) + \sin(\beta - 20°) = 6\cos\beta$$

for $0° < \beta < 90°$. [4]

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87. 9709_w19_qp_22 Q: 8

(i) Express $0.5\cos\theta - 1.2\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation $0.5\cos\theta - 1.2\sin\theta = 0.8$ for $0° < \theta < 360°$. [4]

(iii) Determine the greatest and least possible values of $(3 - \cos\theta + 2.4\sin\theta)^2$ as θ varies. [3]

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88. 9709_w18_qp_21 Q: 3

Solve the equation $\sec^2\theta = 3\,\mathrm{cosec}\,\theta$ for $0° < \theta < 180°$. [5]

89. 9709_w18_qp_22 Q: 7

(i) Use the factor theorem to show that $(2x + 3)$ is a factor of

$$8x^3 + 4x^2 - 10x + 3.$$ [2]

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(ii) Show that the equation $2\cos 2\theta = \dfrac{6\cos\theta - 5}{2\cos\theta + 1}$ can be expressed as

$$8\cos^3\theta + 4\cos^2\theta - 10\cos\theta + 3 = 0.$$ [3]

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(iii) Solve the equation $2\cos 2\theta = \dfrac{6\cos\theta - 5}{2\cos\theta + 1}$ for $0° < \theta < 360°$. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

90. 9709_m17_qp_22 Q: 2

(i) Given that $\tan 2\theta \cot \theta = 8$, show that $\tan^2 \theta = \frac{3}{4}$. [3]

(ii) Hence solve the equation $\tan 2\theta \cot \theta = 8$ for $0° < \theta < 180°$. [2]

91. 9709_s17_qp_21 Q: 5

(i) Express $2\cos\theta + (\sqrt{5})\sin\theta$ in the form $R\cos(\theta - \alpha)$ where $R > 0$ and $0° < \alpha < 90°$, giving the value of α correct to 2 decimal places. [3]

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(ii) Hence solve the equation $2\cos\theta + (\sqrt{5})\sin\theta = 1$ for $0° < \theta < 360°$. [4]

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92. 9709_w17_qp_21 Q: 2

Solve the equation $5\cos\theta(1 + \cos 2\theta) = 4$ for $0° \leqslant \theta \leqslant 360°$. [5]

93. 9709_s16_qp_21 Q: 2

Solve the equation $5\tan 2\theta = 4\cot\theta$ for $0° < \theta < 180°$. [5]

94. 9709_s16_qp_22 Q: 4

(i) Show that $\sin(\theta + 60°) + \sin(\theta + 120°) \equiv (\sqrt{3}) \cos \theta$. [3]

(ii) Hence

(a) find the exact value of $\sin 105° + \sin 165°$, [2]

(b) solve the equation $\sin(\theta + 60°) + \sin(\theta + 120°) = \sec \theta$ for $0° \leqslant \theta \leqslant 180°$. [3]

95. 9709_w16_qp_21 Q: 7

The polynomial $p(x)$ is defined by

$$p(x) = ax^3 + 3x^2 + bx + 12,$$

where a and b are constants. It is given that $(x + 3)$ is a factor of $p(x)$. It is also given that the remainder is 18 when $p(x)$ is divided by $(x + 2)$.

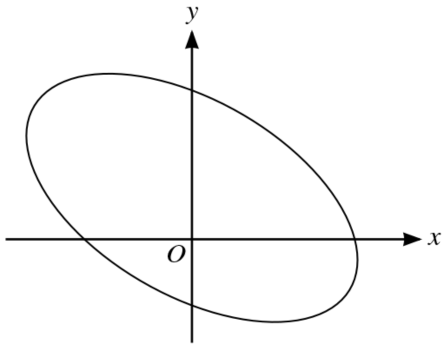
(i) Find the values of a and b. [5]

(ii) When a and b have these values,

(a) show that the equation $p(x) = 0$ has exactly one real root, [4]

(b) solve the equation $p(\sec y) = 0$ for $-180° < y < 180°$. [3]

96. 9709_w16_qp_22 Q: 7



The diagram shows the curve with parametric equations

$$x = 4\sin\theta, \qquad y = 1 + 3\cos(\theta + \tfrac{1}{6}\pi)$$

for $0 \leqslant \theta < 2\pi$.

(i) Show that $\dfrac{dy}{dx}$ can be expressed in the form $k\big(1 + (\sqrt{3})\tan\theta\big)$ where the exact value of k is to be determined. [5]

(ii) Find the equation of the normal to the curve at the point where the curve crosses the positive y-axis. Give your answer in the form $y = mx + c$, where the constants m and c are exact. [5]

97. 9709_w16_qp_23 Q: 7

(i) Express $\sin 2\theta(3\sec\theta + 4\operatorname{cosec}\theta)$ in the form $a\sin\theta + b\cos\theta$, where a and b are integers. [3]

(ii) Hence express $\sin 2\theta(3\sec\theta + 4\operatorname{cosec}\theta)$ in the form $R\sin(\theta + \alpha)$ where $R > 0$ and $0° < \alpha < 90°$. [3]

(iii) Using the result of part **(ii)**, solve the equation $\sin 2\theta(3\sec\theta + 4\operatorname{cosec}\theta) = 7$ for $0° \leqslant \theta \leqslant 360°$. [4]

98. 9709_s15_qp_22 Q: 3

It is given that θ is an acute angle measured in degrees such that

$$2\sec^2\theta + 3\tan\theta = 22.$$

(i) Find the value of $\tan\theta$. [3]

(ii) Use an appropriate formula to find the exact value of $\tan(\theta + 135°)$. [3]

99. 9709_w15_qp_21 Q: 3

(i) Express $8\sin\theta + 15\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$8\sin\theta + 15\cos\theta = 6$$

for $0° \leqslant \theta \leqslant 360°$. [4]

100. 9709_w15_qp_22 Q: 4

The polynomial $\mathrm{p}(x)$ is defined by

$$\mathrm{p}(x) = 6x^3 + 11x^2 + ax + a,$$

where a is a constant. It is given that $(x + 2)$ is a factor of $\mathrm{p}(x)$.

(i) Use the factor theorem to show that $a = -4$. [2]

(ii) When $a = -4$,

(a) factorise $\mathrm{p}(x)$ completely, [3]

(b) solve the equation $6\sec^3\theta + 11\sec^2\theta + a\sec\theta + a = 0$ for $0° \leqslant \theta \leqslant 180°$. [2]

101. 9709_w15_qp_23 Q: 6

(i) Express $(\sqrt{5})\cos\theta - 2\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the value of α correct to 2 decimal places. [3]

(ii) Hence

(a) solve the equation $(\sqrt{5})\cos\theta - 2\sin\theta = 0.9$ for $0° < \theta < 360°$, [4]

(b) state the greatest and least values of

$$10 + (\sqrt{5})\cos\theta - 2\sin\theta$$

as θ varies. [2]